

Comment on the first-order Fermi acceleration at ultra-relativistic shocks

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ABSTRACT

As discussed in several recent papers the first-order Fermi acceleration processes acting at ultra-relativistic shock waves lead to the unique asymptotic spectral index $\sigma_{\gamma \rightarrow \infty} \approx 2.2$. Below, we discuss this result and differences of its various derivations. We also note the not always appropriate astrophysical applications of the discussed mechanism. In particular we point out problems with postulating ultra-relativistic shocks as sources producing the above asymptotic spectra at all $\gamma \gg 1$ or as sources of the gamma ray burst afterglows due to shock accelerated electrons.

Key words: acceleration of particles – relativistic shock waves – X-rays: bursts – gamma-rays: bursts – cosmic rays

1 INTRODUCTION

Ultra-relativistic shock waves suggested to be sources of gamma-ray bursts are also expected by some authors to produce ultra-high-energy (UHE) cosmic ray (CR) particles, with energies in excess of 10^{20} eV. First such models of Waxman (1995) and Vietri (1995) were recently discussed and improved by including essential physical constraints resulting from possible extreme anisotropy of accelerated particles (Gallant & Achterberg 1999, Gallant et al. 1999, Ostrowski 1999, Bednarz & Ostrowski 1999). These papers prove that UHE CR could be produced by such shocks only in the presence of high energy particle reflection from the shock. Unfortunately, efficiency of such reflections is expected to be very low.

The progress was possible due to new results on particle Fermi acceleration in the ultra-relativistic shocks published in a series of papers by Bednarz & Ostrowski (1997a,b,c, 1998 [\equiv B&O’98]; see also Ostrowski 1999, Bednarz 2000a), Gallant et al. (1998) and Gallant & Achterberg (1998, 1999). In section 2 below we shortly compare different approaches to the discussed acceleration process. We point out that spectra generated in realistic shocks with $\gamma \sim 10 - 100$ can be much steeper than the ones obtained in the asymptotic limit of the Lorentz factor, $\gamma \rightarrow \infty$.

The cosmic ray electron spectrum generated in the ultra-relativistic shock is the second problem to be discussed below in section 3. Modelling of gamma ray burst (GRB) afterglows yields often results pointing to the asymptotic spectral index $\sigma \approx 2.2$ for radiating electrons. This fact is sometimes interpreted as observational confirmation of the

proposed theoretical models for ultra-relativistic shock acceleration. Here we critically review this opinion, pointing out that the Lorentz factors considered for shocks in the GRB afterglow phases may be too low to allow for the limiting spectral index. In most cases one should expect formation of much steeper spectra.

Below we use a symbol γ for the shock Lorentz factor, r_g is a particle gyroradius, m_{Fe} , m_p , and m_e are the Iron nucleus mass, the proton mass and the electron mass, respectively; c is the speed of light.

2 ON THE FIRST-ORDER FERMI ACCELERATION AT ULTRA-RELATIVISTIC SHOCKS

The first-order Fermi acceleration process at an ultra-relativistic shock wave involves extreme particle anisotropy at the shock when considered in the upstream plasma rest frame (UPF) and more mild distributions in the shock normal rest frame (SNF) or the downstream plasma rest frame (cf. Begelman & Kirk 1990). Let us consider a particle crossing the shock upstream. Then, in UPF, its momentum is nearly parallel to the shock normal. When the shock Lorentz factor is sufficiently large the particle stays in front of the shock for a very short time. As a result its scattering due to short waves present in front of the shock dominates over its momentum direction variation in the mean magnetic field. A tiny change of particle momentum upstream of the shock allows for its transmission downstream of the shock, where – due to a Lorentz transformation with large γ – its mo-

mentum direction can be changed at a large angle with respect to the original direction before transmission upstream. Such large amplitude angular scattering allows a finite fraction of particles to follow trajectories leading to the successive transmission upstream of the shock. Repeating of the described loops leads to formation of the power law spectrum, with the mean energy gain in one loop comparable to the original energy (B&O'98, Gallant & Achterberg 1999, Gallant et al. 1999a) and the formation of the spectrum with the inclination close to the asymptotic one with the energy spectral index $\sigma_{\gamma \rightarrow \infty} \approx 2.2$. Essentially the same results were obtained within different approaches presented by the above authors and by Guthmann et al. (2000), Kirk et al. (2000) and Vietri (2000).

The work of Bednarz & Ostrowski was based on Monte Carlo simulations of particle transport governed by small amplitude pitch angle scattering. Thus depending on our scattering parameter Δt (a mean time between successive scattering acts) and $\Delta\Omega_{max}$ (the maximum angular scattering amplitude), we were able to model situations with different mean field configurations and different amounts of turbulence. One should note that the mean field configuration downstream of the shock was derived here from the mean upstream field using the appropriate jump conditions. Also, as the particle interaction with the shock discontinuity was derived exactly, the resulting spectra did not depend on possible assumptions about this interaction. Such approach includes also correlations in the process due to the regular part of the magnetic field, irregularities responsible for pitch angle scattering are introduced here as completely random at all length scales. An essential numerical difficulty of such computations lays in the fact that scattering should be performed in the plasma rest frame. Thus, in order to model particle pitch angle diffusion upstream of the shock, with nearly a delta-like angular distribution $\theta \sim \gamma^{-1}$ (θ - particle momentum inclination to the shock normal), an extremely small scattering amplitude should be used, leading to the excessive simulation times. In our simulation we used a relatively 'large' maximum amplitude $\Delta\Omega_{max} = \frac{1}{2}\gamma^{-1}$, but comparison to the results obtained with smaller $\Delta\Omega_{max}$ revealed only insignificant differences of the obtained particle spectrum (as measured at the escape boundary placed at $4r_g$ downstream of the shock). One should note, that due to the above amplitude choice and because in our numerical code the relative particle velocity with respect to the shock instead of the velocity in the plasma rest frame was used in the weighting function in the plasma rest frames, the angular distributions presented by us in B&O'98 are slightly different in comparison to other results (cf. Gallant et al. 1999, Kirk et al. 2000, Vietri 2000).

An analogous, somewhat superficially described pitch angle diffusion modelling appended considerations of Gallant et al. (2000) and Gallant et al. (1998, 1999). If we understand properly, they considered a highly turbulent conditions near the shock leading to the particle pitch angle diffusion *with respect to the shock normal*, i.e. the regular part of the magnetic field was neglected. These computations gave essentially the same spectral indices as the asymptotic one derived by B&O'98. Also, in a variant of this model with uniform magnetic field upstream of the shock and fully chaotic turbulent field downstream, the resulting spectral index did not varied substantially. The physical con-

tent of this qualitative model is substantially different from the Bednarz & Ostrowski's one. In particular it neglects influence of the uniform field (or long wave length perturbations with $\lambda > r_g$) resulting in magnetic field correlations at both sides of the shock. Thus, for example, if the amplitude of the magnetic field turbulence is limited, it can not reproduce spectrum steepening at intermediate Lorentz factors in the presence of oblique magnetic fields (cf. B&O'98, Begelman & Kirk 1990). Also, in the model of Bednarz & Ostrowski the asymptotic spectrum hardly depends on existence or non-existence of the downstream turbulence, the essential ingredient of the acceleration process is the short wave pitch angle scattering upstream of the shock. The both models describe essentially the same physical situation only in the limit of highly turbulent medium.

An excellent discussion of the acceleration process presented by Gallant & Achterberg (1998, 1999) was based on a simple turbulence model. In their approach highly turbulent magnetic configurations were assumed upstream and downstream of the shock, idealized as cells filled with randomly oriented uniform magnetic fields. With such approach particles crossing the shock enter a new cell with the randomly selected magnetic field configuration. Thus, there always occur configurations allowing some particles crossing downstream to reach the shock again and to form the power law spectrum. In this model there is no need for the upstream magnetic field perturbations and a model with the uniform upstream field yields also the power law spectrum.

Two quasi-analytic approaches to the considered acceleration process were recently presented by Kirk et al. (2000, see also Guthmann et al. 1999) and Vietri (2000). Both attempt to solve the Fokker-Planck equation describing particle advection with the general plasma flow and the small amplitude scattering of particle pitch angle as measured with respect to *the shock normal*. Kirk et al. modified the Kirk & Schneider (1987) series expansion approach which allows to treat the delta-like angular distributions upstream of the shock. An analytically more simple Vietri approach applies convenient ansatz'es for the anisotropic upstream and downstream particle distributions, resembling the Peacock's (1981) approach to 'ordinary' relativistic shocks. Both methods confirm the results of the earlier numerical modelling. A deficiency of the above semi-analytic approach is its inability to treat situations with mildly perturbed magnetic fields.

As a conclusion from this section let us point out that different approaches to the cosmic ray Fermi acceleration at relativistic shocks yield consistent estimates of the asymptotic spectral index $s \approx (2.2, 2.3)$. Only B&O'98 modelling allows to treat – in a highly idealized way – conditions with medium amplitude perturbations of the magnetic field. In such conditions particle spectra are expected to be much steeper at medium Lorentz factors, approaching the asymptotic spectral index only in the limit of $\gamma \rightarrow \infty$.

3 ON ELECTRON SPECTRA

Nearly all the above mentioned papers on acceleration at ultra-relativistic shocks make a reference of the derived spectra to the ones observed (modelled) in GRB afterglows. Coincidence of the obtained spectral indices, also for the case

of radiation from the Crab relativistic wind terminal shock, is claimed as nearly a proof of acting the Fermi acceleration processes at ultra-relativistic shocks. However, a more detailed insight into the considered process presents several objections to such interpretation.

The application of the first-order Fermi acceleration process at the shock wave requires that particle mean free paths normal to the shock, and in particular particle gyroradia, are much larger than the dissipative thickness of the shock, d_{diss} . In the GRB models involving a shock propagating into the ordinary interstellar medium protons and helium nuclei define this length scale. Thus, for ultra-relativistic shocks the discussed theory with the resulting 'universal' spectral index can be applicable only for electrons with the energy $> \gamma m_p c^2 / \sqrt{2} \approx \gamma \text{ GeV}$. Downstream of the shock such electrons radiate at frequencies $\nu \approx 1.4 B_{\mu G} \gamma_e^2 \approx 3 \times 10^6 B_{\mu G} \gamma^3 \text{ Hz}$ if the magnetic field equals to a few μG upstream magnetic field, $B_{\mu G}$, multiplied by a compression factor $\sim \gamma$ (higher frequencies are involved if the downstream equipartition fields are considered). As this radiation originates in the plasma rest frame, the observed in GRB afterglows frequencies may be a factor of γ higher, above $\sim B_{\mu G} \gamma^4 \text{ MHz}$ (cf. Gallant et al. 1999b). It may be even more difficult to apply such mechanism for electrons at the Crab pulsar termination wind. In this case the involved Lorentz factor can reach values $\sim 10^6$ and the heavy component upstream of the shock may consist of iron nuclei, yielding a lower energy limit for the first-order Fermi accelerated particles at energies $\sim 10^8 \text{ GeV}$ (!). Below this energy particles are expected to be accelerated in different processes. E.g. in the Hoshino et al. (1993) modelling of the terminal shock of the Crab pulsar wind the power law electron energy spectra were obtained in the range $(\gamma m_e c^2, \gamma m_F c^2)$ with the energy spectral indices in the range close to 2.0, compatible with observations (Gallant & Arons 1994; see also Arons 2001). One could also make a reference to a number of papers discussing acceleration at GRB shocks with rough equipartition arguments and obtaining (more exactly: postulating) similar spectral inclinations. A more promising conditions for application of the discussed theory provide the shocks propagating in pure electron-positron pair plasma.

Another problem for generation of asymptotic spectra is the shock Lorentz factor diminishing at the afterglow phase of GRB remnant evolution. As show simulations of B&O'98 a presence of regular component of the magnetic field leads in such shocks to generation of very steep spectrum. Thus, even if sufficiently high energy electrons exist near the shock the main acceleration process can be a simple super-adiabatic compression at the shock, as discussed by Begelman & Kirk (1990). Basing on simulations one should rather expect the temporal steepening of the particle spectrum generated by the slowing down shock propagating in mildly perturbed magnetic field. As seen from figures in B&O'98 this steepening may appear at quite large shock Lorentz factors, depending on configuration of the mean magnetic field and the turbulence amplitude.

4 CONCLUSIONS

We discussed the first-order Fermi acceleration process acting at ultra-relativistic shocks and pointed out several limitations for the application of this process. In particular acceleration of the cosmic ray electrons of interest for explaining the GRB afterglows can be in our opinion caused by another processes. In a case of the slowing down, but still ultra-relativistic shock wave the resulting particle spectrum is expected to steepen with time. Also, as mentioned in the first section, cosmic ray acceleration to ultra high energies by reflections from ultra-relativistic shocks is expected to be weakly efficient.

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